INTRODUCTION TO MATHEMATICAL DIAGNOSTICS
I. THEORETICAL BACKGROUNDS

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RESUME

Technical state of the dynamical system can be identified by the means of the changes of the so called (measurable) external and (non-measurable) internal parameters. One of the main goals of mathematical diagnostics is the determination of momentary location of the examined system and to forecast of the direction and velocity of its movement in this multidimensional state space. The paper shows theoretical backgrounds and basic methodology of mathematical diagnostic modeling of technical systems.

Keywords: *modeling, mathematical models, diagnostics*

1. INTRODUCTION

During operation of technical systems and equipment they are going through continuous and cumulative changes. By using, their technical state generally changes in the negative sense, while during the service or repair, it changes positively. The technical state of the system can be identified by the means of the changes of the so called (measurable) external and (non-measurable) internal parameters. Therefore momentary technical state of the examined system can be given mathematically as a point of multidimensional state space defined by external and internal parameters.

One of the main goals of mathematical diagnostics is the determination of momentary location of the examined system and to forecast of the direction and velocity of its movement in this multidimensional state space.

Practically, internal parameters cannot be determined directly because of technical and economic hindrances. Their values, changes and changing velocities can be estimated by using a state-estimation method based on mathematical modeling. Knowing the momentary technical state and its changing velocity, the optimal operational strategy and needed service work can be decided [7].

The other aim of mathematical diagnostics is investigation and description of system behavior in case of different technical situations (for example prohibited duties) and environmental condition. Such mathematical diagnostic methods are the following ones: sensitivity test, correlation family test, and mathematical modeling of prohibited duties of the examined system. Using mathematical models, influences of the manufacturing anomalies can be investigated and characterized as well.

This paper will show the basics and basic methodology of mathematical diagnostic modeling of technical systems. The paper will be organized as follows: Section 2 shows the main classifications of mathematical models. Section 3 words their creation. Section 4 presents theoretical setting-up of the mathematical models. Section 5 shows methods of linearization. Section 6. states few non-linear mathematical solution processes.

2 THE MATHEMATICAL MODELS

The model is a simplified copy of the real system which emphasizes its important characteristics and neglects secondary characters are not determinant ones from the point of view of the investigation. The abstracted model can be analog, homologue and mathematical one.

The homologue model is similar to the real system geometrically. For example, wind-tunnel models of airfoils or aircraft are homologue models. The analog model does not need to be similar to the real system, but its principle has to be analog with the process occurring on the real modeled system. As a rule, an analog computer means an analog model [10].

In the engineering practice the most widespread model is the mathematical one. Nowadays, the computer technology and numerical-mathematical methods are developing quickly and at the same time diagnostics methods that are based upon the mathematical modeling become important and developing part of the engineering practice.

The mathematical model gives the most concise characterization of an occurrence [2]. The mathematical model is the mathematical equation or system of equations which describes the internal principles of the process occurring on the system from the point of view of the given investigation.

On the basis of the characters of mathematical equations which describe the behavior of the system or their determination, the following mathematical models can be distinguished in pairs [6]:

Static - Dynamic

The mathematical model will be a static one if the state of the system can be described by algebraical equations or differential equations which do not contain derivatives with respect to the time. As a rule, they are called as stationary or steady models.

The dynamic (non-stationary) mathematical models describe the changes of the system parameters depending on the time. They can be vulgar or partial differential equations. It is possible that the changes of the system parameters can be described by not only on a time interval, but on a transformed domain chosen practically.

Linear - Non-linear

The linear models consist of only variables or their derivatives multiplied - in general stationary - coefficients. A linear mathematical model can be a linear or linearized equation or a system of equations.

The non-linear mathematical models are free from the requirement mentioned above. At least one of the equations which describe process occurs in the system is non-linear.

For simpler solving and investigation, the non-linear mathematical models can be transformed into linear ones (see Chapter 5).

Continuous parameter - Discrete parameter

In case of continuous parameter (continuous time) models the dependent variables can have values at every moment of the investigated time interval continuously.

The mathematical model is a discrete parameter (discrete time) one if its dependent variables can only have some value at certain moments of the investigated time.

As a rule, continuous time processes are modeled by discrete time models when the equations are solved numerically by any time-shifting.

Continuous state-space - Discrete state-space

If the variables can have all value continuously, the mathematical model is called a continuous state-space one.

A mathematical model is considered to be of a discrete state-space one if the values of the dependent variables constitute a count finite or non-finite set.

For easier solving, continuous state-space processes can be transformed to discrete state-space one by discretization.

Deterministic - Stochastic

In case of a deterministic model, the dependencies of output parameters on input parameters can be described unambiguously in some time internal or domain of the independent variables.

Stochastic (random) mathematical model includes random functions which can describe interdependencies between dependent and independent variables of the examined system.

Obviously the list mentioned above of the kinds of the mathematical models is not complete, of course. Mathematical model realized in the engineering practice means the synthesis of the above mentioned ones.

3. CREATING MATHEMATICAL MODEL

The logic and the type of the modeling are determined by answers for the following question:

- ✈ What are the main goals of the investigation based on mathematical modeling?
- ✈ How can the mathematical model be set up?
- ✈ How can the mathematical model be checked and qualified?
- ✈ What is the most optimal strategy of the collection of the missing data?
- ✈ How can the non-linearity be solved?
- ✈ What are the economic and technical requirements?
- ✈ Can you experiment with the real system continuously for the checking of the model?

- How can the mathematical model be simplified?
- What is the most optimal model for the investigation of the real system?

The setting up of a real model can be carried out based upon the logical scheme that Figure 1. shows. The possible errors are written in italics [8].

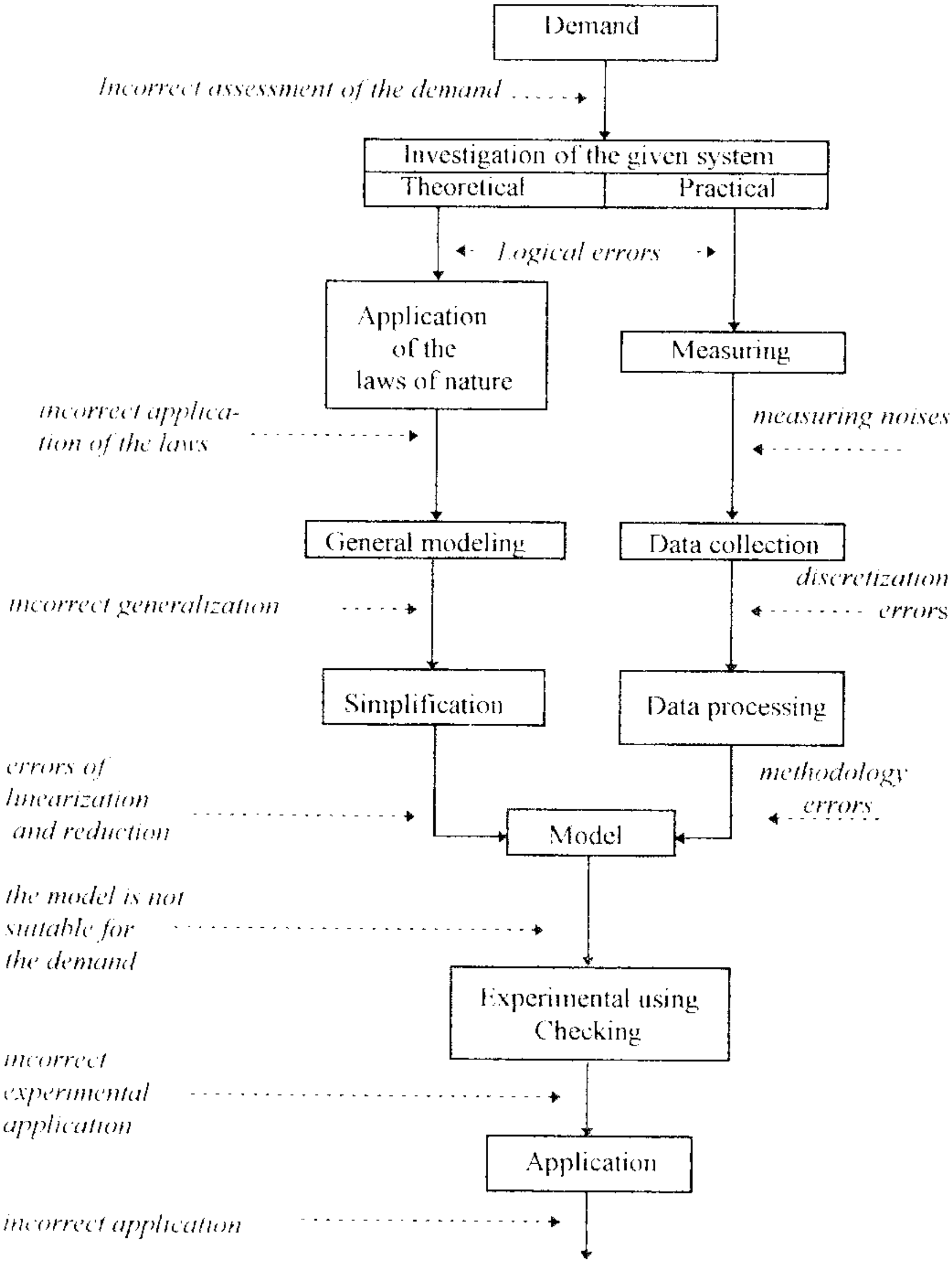

Fig.1. Logical Scheme of Modeling

In 1979 the Technical Committee on Model Credibility of the Society for Computer Simulation developed a diagram identifying the primary phases and activities of modeling and simulation [4]. Figure 2 shows that analysis is used to construct a conceptual model of reality. Programming converts the conceptual/mathematical model into a computerized model. Then computer simulation is used to simulate reality. Although simple and direct, the diagram clearly captures the relationship of two key phases of modeling and simulation to each other and to reality. The diagram also includes the activities of model qualification, model verification, and model validation.

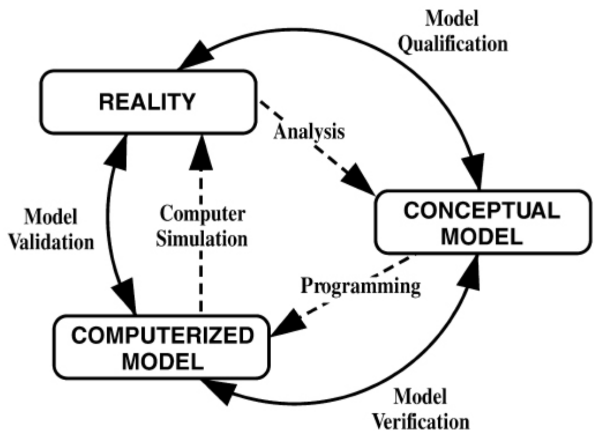

Fig. 2. View of modeling by the Society for Computer Simulation [4]

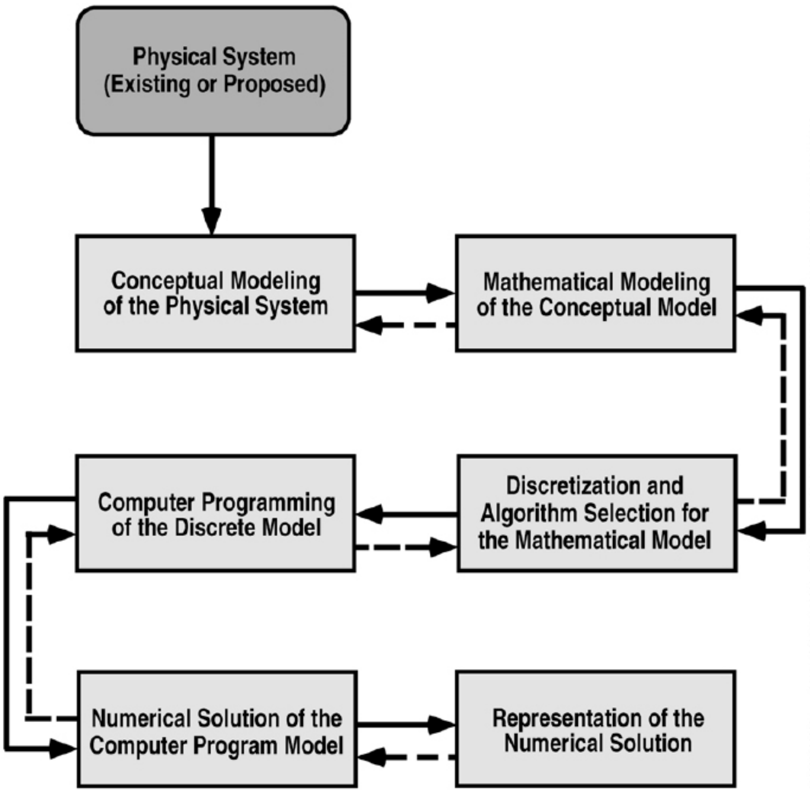

Fig. 3. Proposed Phases for Mathematical Modeling [4]

Figure 3 depicts our representation of the phases of modeling and simulation. The phases represent collections of activities or tasks required in a large-scale simulation analysis, particularly models given by differential equations and their numerical solution. The ordering of the phases implies an information and data flow that indicates which activities are likely to impact decisions and methodology occurring in later phases. However, there is significant feedback and interaction between the phases, as indicated by the dashed lines in the figure.

4. THE SETTING-UP OF THE MATHEMATICAL MODEL

In this chapter the basic methodology of mathematical modeling of technical systems will be shown. Since the author's main goal is to demonstrate it for engineers who like to use mathematical models during their work, the method will be shown in the case of the example shown by Figure 4.

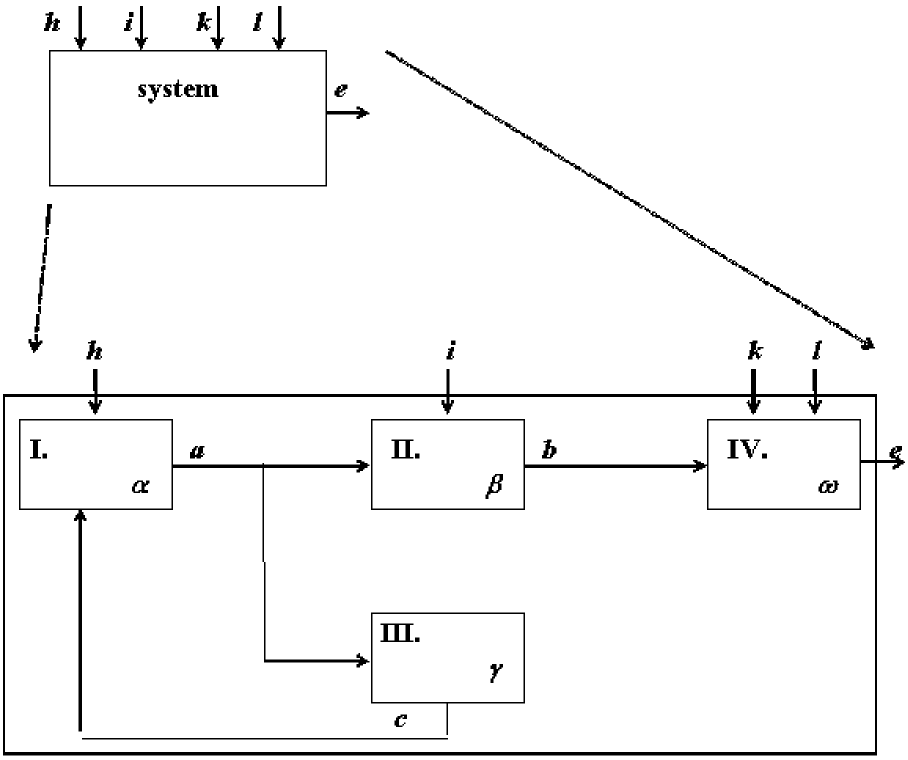

Fig. 4. Block and Block-Diagram of the System

The setting up of mathematical model should start with splitting up the investigated system into its functional units. The Figure 4. demonstrates this step of the modeling. Now it is a very important question that which part of the system is necessary for modeling of its investigated regime. For example in case of a stationary regime of pneumatic and hydraulic systems, the filters are not important equipment. Because of in this case the pressures in the system have been equalized. But during their non-stationary regimes the filters have very important function as chokes which influence

the change of pressures in several chambers of the system in the function of time.
These above determined units should be examined and the interdependencies between their input and output parameters should be established mathematically. In the technical practice, the mathematical model can be written basically in two ways:

- **WHITE BOX METHOD;**

The model should be written by analytical equation on the basis of scientific knowledge. In this case one should use physical rules which depict processes occurring in the investigated equipment. Using white box method, you have to know the working principle and the nominal (designed) values of all internal parameters of given system unit.

- **BLACK BOX METHOD.**

The model is written by analyzing of the output parameters responded to the given input ones. This method should be used if the physical processes occurring in the parts of the system and internal structures of equipment are not known. In this case the mathematical model should be set-up by the investigation of the behavior of the real system. For example one of the black box methods is the dimensional analysis.

The equations mentioned above form the mathematical model of the system. For example (for following demonstrations) this model can be written in the case mentioned above, that is the mathematical model of the system see Figure 4. :

- Equipment I. :

$$a = hc\alpha \tag{1}$$

- Equipment II. :

$$b = a + i + \beta \tag{2}$$

- Equipment III. :

$$c = a\gamma^{\kappa} \tag{3}$$

- Equipment IV. :

$$e = \frac{2kl\omega^{\frac{\kappa-1}{\kappa}}}{b} \tag{4}$$

or in a simpler way:

$$f(\underline{y}) = g(\underline{x}) \tag{5}$$

where:

$\underline{x}$ - vector of dependent parameters:

$$\underline{x}^T = \begin{bmatrix} a & b & c & e \end{bmatrix} \tag{6}$$

$\underline{y}$ - vector of independent parameters:

$$\underline{y}^T = [h \quad \alpha \quad i \quad \beta \quad \gamma \quad k \quad l \quad \omega] \quad . \tag{7}$$

The elements of the vectors are parameters of the example system — see Figure 1. The α ; β ; γ and ω are internal parameters of equipment (for example stiffness of spring or flown area). The a ; b ; c ; e ; h ; i ; k and l are the input and output parameters of the given system and equipment (for instance pressures, mass flow and power).

5. SETTING-UP OF THE LINEAR MATHEMATICAL DIAGNOSTIC MODELS

For setting up a linear diagnostic model, the mathematical model which is basically a non-linear system of equations should be linearized. For linearization, the following methods can be used:

- LOGARITHMIC LINEARIZATION;
- DIRECT DIFFERENTIATION;
- TAYLOR SERIES;
- LIE-MAGNUS SERIES.

In the following only the logarithmic linearization will be depicted in detail, because this method is not well-known.

5.1. The Logarithmic Linearization

Using the logarithmic linearization, firstly, the natural logarithm (to e base) of both sides of the general non-linear equation

$$y = f(x_1; x_2; \ldots x_n) \tag{8}$$

should be formed:

$$\ln y = \ln f(x_1; x_2; \ldots x_n) \quad . \tag{9}$$

As the next step, the total differential of the latter one should be formed, using the basic differential quotient of the natural logarithm:

$$(\ln \eta)' = \frac{1}{\eta} \quad , \tag{10}$$

and the rule of derivation of the function of functions. We introduce the equation

$$\frac{d\beta_i}{\beta_i} \cong \frac{\Delta\beta_i}{\beta_i} = \delta\beta_i \tag{11}$$

and substitute for the equation given above formally. Then one get the equation

$$\delta y = K_1\delta x_1 + K_2\delta x_2 + \ldots K_n\delta x_n \quad , \qquad 2)$$

which describes the relation between relative changes of different variables of the original equation (8) by a linear form.

See the logarithmic linearization of the model set-up above:

✈ In case of equation (1) the natural logarithm of both sides:

$$a = hc\alpha \quad \Rightarrow \quad \ln a = \ln(hc\alpha) = \ln h + \ln c + \ln\alpha \quad . \tag{13}$$

The total differential:

$$\frac{da}{a} = \frac{dh}{h} + \frac{dc}{c} + \frac{d\alpha}{\alpha} \quad , \tag{14}$$

then:

$$\delta a = \delta h + \delta c + \delta\alpha \quad . \tag{15}$$

✈ In case of equation (2) :

$$b = a + i + \beta \quad \Rightarrow \ln b = \ln(a + i + \beta) \quad . \tag{16}$$

The total differential:

$$\frac{db}{b} = \frac{1}{a+i+\beta}da + \frac{1}{a+i+\beta}di + \frac{1}{a+i+\beta}d\beta \quad . \tag{17}$$

In this case every term should be multiplied by $\frac{x_i}{x_i}$:

$$\frac{db}{b} = \frac{a}{a(a+i+\beta)}da + \frac{i}{i(a+i+\beta)}di + \frac{\beta}{\beta(a+i+\beta)}d\beta \tag{18}$$

$$\delta b = \frac{a}{a+i+\beta}\delta a + \frac{i}{a+i+\beta}\delta i + \frac{\beta}{a+i+\beta}\delta\beta \tag{19}$$

✈ In case of equation (3):

$$c = a\gamma^{\kappa} \quad \Rightarrow \ln c = \ln a + \kappa \ln \gamma \tag{20}$$

$$\frac{dc}{c} = \frac{da}{a} + \kappa \frac{d\gamma}{\gamma} \tag{21}$$

then:

$$\delta c = \delta a + \kappa \delta \gamma \quad . \tag{22}$$

✈ In case of equation (4):

$$e = \frac{2kl\omega^{\frac{\kappa-1}{\kappa}}}{b} \quad \Rightarrow \ln e = \ln 2 + \ln k + \ln l + \frac{\kappa-1}{\kappa} \ln \omega - \ln b \tag{23}$$

the derivative of a constant (*ln2*) equals zero, that is:

$$\frac{de}{e} = \frac{dk}{k} + \frac{dl}{l} + \frac{\kappa-1}{\kappa}\frac{d\omega}{\omega} - \frac{db}{b} \tag{24}$$

in other form:

$$\delta e = \delta k + \delta l + \frac{\kappa-1}{\kappa}\delta\omega - \delta b \quad . \tag{25}$$

The logarithmic linearization should be used in case of thermodynamic models and equations because of their exponential terms.

5.2. Direct Differentiation

Using direct differentiation, as a first step, the total differential of both sides of the initial equation

$$y = f(x_1; x_2; \ldots x_n) \quad , \tag{26}$$

should be formed:

$$dy = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_1} dx_1 + \ldots + \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_n} dx_n \quad . \tag{27}$$

Then both sides of the last equation should be multiplied by same sides of the general equation and all elements should be multiplied by $\frac{x_i}{x_i}$:

$$\frac{dy}{y}=\frac{\partial f(x_1;x_2;\dots x_n)}{\partial x_1}\frac{x_1}{f(x_1;x_2;\dots x_n)x_1}dx_1+\dots$$
$$\dots+\frac{\partial f(x_1;x_2;\dots x_n)}{\partial x_n}\frac{x_n}{f(x_1;x_2;\dots x_n)x_n}dx_n \tag{28}$$

Using the coefficients:

$$K_i=\frac{\partial f(x_1;x_2;\dots x_n)}{\partial x_i}\frac{x_i}{f(x_1;x_2;\dots x_n)} \tag{29}$$

and equation (11), the following linear system can be achieved:

$$\delta y=K_1\delta x_1+\dots+K_n\delta x_n \quad . \tag{30}$$

This method is basically suggested if the general equation cannot be decomposed to multipliers.

5.3. The Taylor (Lie-Magnus) Series

In this case TAYLOR-series of the general equation

$$y=f(x_1;x_2\dots x_n) \tag{31}$$

should be developed:

$$y+\Delta y=f(x_1;x_2\dots x_n)+\sum_{i=1}^{\infty}\frac{\partial^i f(x_1;x_2\dots x_n)}{\partial x_1^i}\frac{1}{i!}\Delta x_1^i+\dots \tag{32}$$

and its more than first-order terms have to be neglected

$$\Delta y=\frac{\partial f(x_1;x_2\dots x_n)}{\partial x_1}\Delta x_1+\dots \quad . \tag{33}$$

Then its both sides should be divided by the same side of the initial equation:

$$\frac{\Delta y}{y}=\frac{\partial f(x_1;x_2\dots x_n)}{\partial x_1}\frac{1}{f(x_1;x_2\dots x_n)}\frac{x_1}{x_1}\Delta x_1+\dots \quad . \tag{34}$$

Using equations (11) and (29) the following equation can be achieved:

$$\delta y=K_1\delta x_1+\dots+K_n\delta x_n \quad . \tag{35}$$

This linearization method can be used if the general equation can be derivable any times. The linearization using by TAYLOR series is applied basically in flight-mechanical investigations.

The LIE-MAGNUS series method is the so called matrix-form version of the TAYLOR series one that can be used for linearization of non-linear system of equations. This method uses the derivation matrix to derive the initial system of equations [5].

5.4. The Diagnostic Matrix

The equations (15); (19); (22) and (25) form a system of equations that is the linear (linearized) mathematical diagnostic model of system shown by Figure 4.

The linear system of equations achieved in this way describes interdependencies between relative changes of independent ($\delta\underline{x}$) and dependent ($\delta\underline{y}$) parameters from the point of view of the given investigation - see equations (6) and (7). This model can be written in the following matrix formula:

$$\underline{\underline{A}}\delta\underline{y} = \underline{\underline{B}}\delta\underline{x} \quad , \tag{36}$$

where $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are coefficient matrices of external and internal parameters of the investigated system.

In case of system shown by Figure 1 the coefficient matrices are:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ -\dfrac{a}{a+i+\beta} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} , \tag{37}$$

$$\underline{\underline{B}} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{i}{a+i+\beta} & \dfrac{\beta}{a+i+\beta} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & \dfrac{\kappa-1}{\kappa} \end{bmatrix} \tag{38}$$

Using the

$$\underline{\underline{D}} = \underline{\underline{A}}^{-1}\underline{\underline{B}} \tag{39}$$

diagnostic matrix, the equation

$$\delta\underline{y} = \underline{\underline{D}}\delta\underline{x} \tag{40}$$

can be used for diagnostic investigations that will be shown in the following chapters.

6. SOLUTIONS OF NON-LINEAR MATHEMATICAL MODELS

The non-linear mathematical model set-up in Chapter 4. should be solved. Because a real technical system is a multiple-unit one its mathematical model is a system of equations. In case of non-linear systems of equations few basic methods (for instance the chord-method) of solution of non-linear system cannot be used. In the technical practice the following methods are basically used to solve a non-linear system of equations:

- ✈ NEWTON-RAPSHON METHOD;
- ✈ GRADIENT METHOD;

6.1. The Newton-Rapshon Method

To solve the non-linear system of equations

$$\begin{aligned} f_1(x_1;\ldots;x_n) &= 0 \\ &\vdots \\ f_n(x_1;\ldots;x_n) &= 0 \end{aligned} \quad , \tag{41}$$

suppose that x_{11} ; x_{21} … x_{n1} are its an approximate solution. In this case:

$$\Delta f_i = f_i(x_1;\ldots;x_n) \quad . \tag{42}$$

Then the Taylor series of functions should be developed and its more then first-order terms have to be neglected:

$$\begin{aligned} \frac{\partial f_1}{\partial x_1}\,\Delta x_1 \;\ldots\; \frac{\partial f_1}{\partial x_n}\,\Delta x_n &= \Delta f_1 \\ &\vdots \\ \frac{\partial f_n}{\partial x_1}\,\Delta x_1 \;\ldots\; \frac{\partial f_n}{\partial x_n}\,\Delta x_n &= \Delta f_n \end{aligned} \quad . \tag{43}$$

The linear system of equations (43) should be solved to vector $\Delta \underline{x}$ by any method and using this solution the next approximate solution is going to be:

$$\begin{aligned} x_{1,j+1} &= x_{1,j} + \Delta x_1 \\ &\vdots \\ x_{n,j+1} &= x_{n,j} + \Delta x_n \end{aligned} \quad . \tag{44}$$

If all x_i fulfill the following inequality

$$\left| x_{i,j} \right| < \varepsilon \quad , \tag{45}$$

the solution can be accepted, where:

ε —the acceptable deviation.

6.2. The Gradient Method

The essence of this method is that changing the value of the given scalar - vector function is studied in normal direction of the level surface in the *n*-dimensional space determined by the dependent variables of the function.

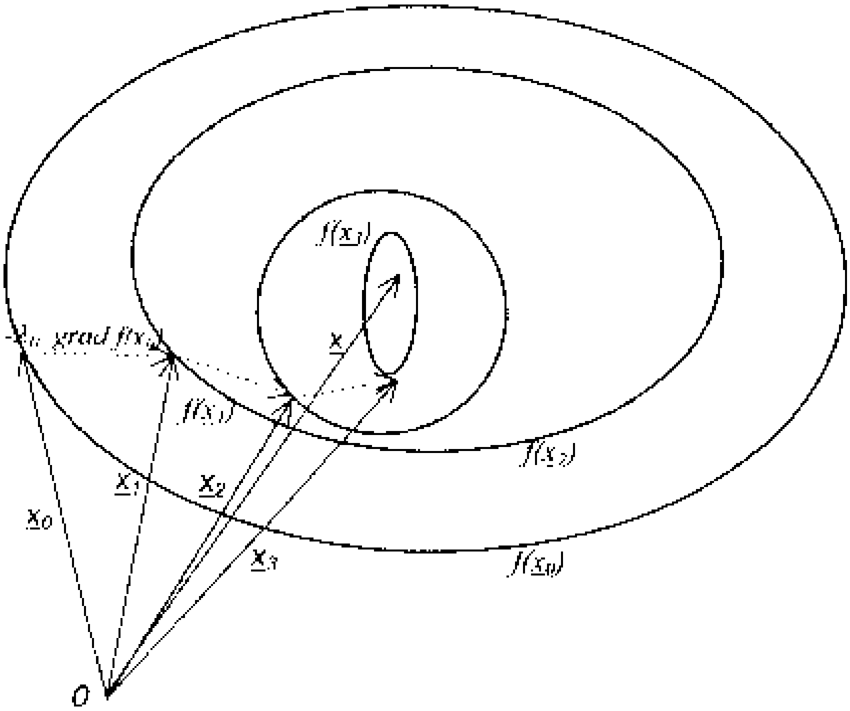

Figure 5. The Gradient Method

Starting from point $\underline{x}_0$ that means the zero-th approximate value belongs to level surface $f(\underline{x}) = f(\underline{x}_0)$, one should determine the normal direction that is the gradient of the surface at point $\underline{x}_0$. Along this gradient vector, you should determine the first approximate value $\underline{x}_1$ and its level surface $f(\underline{x}) = f(\underline{x}_1)$. Then you should determine point (and approximate value) $\underline{x}_2$ and its surface $f(\underline{x}) = f(\underline{x}_2)$ and so on …

Because

$$f(\underline{x}_0) > f(\underline{x}_1) > f(\underline{x}_2) > \ \dots \quad , \tag{46}$$

you can get a point where the value of function $f(\underline{x})$ is the minimal. This vector is the solution of the given equation.

The gradient, which is the vector that shows the direction and intensity of the increasing of the scalar - vector function $f(\underline{x})$, can be determined by equation

$$grad f(\underline{x}) = \nabla f(\underline{x}) \quad , \tag{47}$$

where:

∇ - Hamilton (nabla) operator:

$$\nabla = \begin{bmatrix} \dfrac{\partial}{\partial x_1} \\ \dfrac{\partial}{\partial x_2} \\ \vdots \\ \dfrac{\partial}{\partial x_n} \end{bmatrix} \quad . \tag{48}$$

The gradient method is illustrated by Figure 5 and its iteration equation is

$$\underline{x}_{i+1} = \underline{x}_i - \lambda_i \nabla f(\underline{x}_i) \quad , \tag{49}$$

where:

λ_i - multiplier of i-th iteration step $i = 1 ; 2 ; \ldots$

7. CLOSING REMARKS

The paper — as the first part of a series — has shown basic classifications of mathematical models, their creation process, theoretical tasks of mathematical models' setting-up. Using an academic case the setting-up method has been shown. The last parts of paper showed methods of linearization and few non-linear mathematical solution processes. The series will be continued.

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BEVEZETÉS A MATEMATIKAI DOAGNOSZTIKÁBA I. ELMÉLETI ALAPOK

Egy technikai rendszer üzemeltetése során annak műszaki állapota, a nem-mérhető, úgy nevezett belső paraméterei véletlenszerűen változnak. Így a rendszer pillanatnyi állapotát a belső paraméterek által meghatározott többdimenziós tér egy pontjaként jellemezhető. A matematikai diagnosztika feladata a fent említett térben a rendszer pillanatnyi helyzetének, illetve mozgási irányának és sebességének meghatározása a vizsgált rendszer matematikai modelljének felhasználásával.

A tanulmány a matematikai modellek osztályozási lehetőségeit, a matematikai modell felállításának lépéseit elvben és egy példán keresztül, a linearizálási módszerek, illetve a nem-lineáris modellek megoldásának néhány módszerét mutatja be röviden.